ELSEVIER
Exergy, an International Journal 2 (2002) 152–158
exergy
An International Journal
www.exergyonline.com

How much work is lost in an irreversible turbine?

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Received 5 July 2001; accepted 1 March 2002

Abstract

The question of how much work is lost in an adiabatic turbine due to its irreversibilities finds different answers when discussed on basis of the isentropic efficiency, or with the exergy method. In this contribution, we seek to clarify why the two viewpoints lead to quite distinct results for the lost work. In particular, we discuss how the "reversible work" of the exergy method could be realized and how to recover the "recoverable work of friction". The difference between both approaches is explained.

1. Introduction

Many researchers and practicing engineers agree that exergy analysis is a powerful tool for assessing the thermodynamic efficiencies and losses of systems and processes [1–6]. Consequently, exergy methods are used in some industries. However, many industries use exergy analysis only sparingly or not at all.

One of the present authors, in considering reasons for this lack of broader acceptance, recently wrote [7], "those who choose not to utilize exergy often do so for several reasons. Some find … the results difficult to interpret and understand." In addition, users often consider many facets of exergy analysis to be confusing. If exergy methods are to be more widely accepted and adopted by industry, these issues must be addressed. Confusion and lack of understanding can be addressed, in large part, through better education and clearer presentation.

Although many researchers are trying to alleviate such problems, confusion nevertheless still exists about some aspects of exergy analysis. This statement is sometimes true even for relatively simple processes and devices. One such device is an adiabatic turbine, which is the focus of this article.

In conventional thermodynamics, the second law is used to assess the merit of a turbine and leads to the common isentropic efficiency, among other measures. With exergy methods, more than one type of efficiency can be defined and used. A common definition of turbine exergy efficiency, which has often been used by others, is considered here.

Significant confusion can arise on the part of users because these turbine efficiencies are different, and sometimes the magnitudes of the differences are significant. For example, Kotas [5] notes for an expansion process that the isentropic efficiency differs from the rational efficiency, which is based on exergy, and he describes the differences. Also, Moran and Shapiro [6] obtain different values for isentropic and exergy efficiencies in examples. But explanations regarding which of these efficiencies are more useful or meaningful, and how they should be interpreted, are lacking. The confusion that can consequently arise can leave engineers unsure of what actions are needed to improve efficiency. Worse still, this situation can be misleading, and can cause inappropriate actions regarding efficiency improvement to be taken.

Other researchers have also recognized some of the difficulties related to second law-based and other definitions of efficiencies for turbines, and several articles on the topic have recently been published (e.g., [8–10]).

In this article, we consider isentropic and exergy efficiencies for an adiabatic turbine and explain the differences between them. Furthermore, we discuss the implications of each, in terms of actions that may be implied from them to improve efficiency. Our objectives are to help eliminate the confusion that can arise when isentropic and exergy efficiencies are considered for a turbine, and to clarify the meanings and advantages of each. It is hoped that the results will help

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1164-0235/02/$ – see front matter
PII: S1164-0235(02)00068-7

Nomenclature

c_p	specific heat at constant pressure	$J\cdot kg^{-1}\cdot K^{-1}$
C	flow velocity	$m\cdot s^{-1}$
h	specific enthalpy	$J\cdot kg^{-1}$
k	isentropic exponent	
$\dot{m}$	mass flow	$kg\cdot s^{-1}$
p	pressure	bar
p_0	reference environment pressure = 1.01325 bar	
$\dot{Q}_0$	heat flow rate at temperature T_0	W
$\dot{Q}_k$	heat flow rate at temperature T_k	W
R	gas constant	$J\cdot kg^{-1}\cdot K^{-1}$
s	specific entropy	$J\cdot kg^{-1}\cdot K^{-1}$
$\dot{S}_{gen}$	entropy production rate	$W\cdot K^{-1}$
T, T_k	temperature	K
T_0	reference temperature = 298.15 K	
T_{2s}	end temperature of isentropic expansion	K
w_T	specific turbine work	$J\cdot kg^{-1}$
$w_{T,s}$	specific work of isentropic turbine	$J\cdot kg^{-1}$
$w_{T,rev}$	specific work (exergy analysis)	$J\cdot kg^{-1}$
$\dot{W}$	work rate (power)	W
$\dot{W}_{lost}$	lost power	W
$\dot{W}_{lost}^{exergy}$	lost power according to exergy analysis	W
$\dot{W}_{rev}$	reversible power	W

Greek symbols

η_T	isentropic turbine efficiency	
η_{II}	second-law efficiency	
σ	specific entropy production	$J\cdot kg^{-1}\cdot K^{-1}$
ψ	specific flow exergy	$J\cdot kg^{-1}$

engineers utilize the different turbine efficiencies more effectively, and facilitate greater use of exergy methods in appropriate and beneficial ways.

2. Exergy and reversible work

Let us introduce exergy following Bejan [1]. Since we are interested only in turbines, we restrict ourselves to stationary processes in open systems of constant volume with one inlet and one exit. Then, first and second law of thermodynamics can be written as

$$\dot{m}\left(h_2 - h_1 + \frac{1}{2}\left(C_2^2 - C_1^2\right)\right) = \sum_{k=0} \dot{Q}_k - \dot{W}$$
$$\dot{m}(s_2 - s_1) - \sum_{k=0} \frac{\dot{Q}_k}{T_k} = \dot{S}_{gen} \geqslant 0 \tag{1}$$

We ask for changes in the process, in order to maximize the power (i.e., work per unit time) $\dot{W}$. Note, that if $\dot{W} > 0$, maximizing is tantamount of increasing the power output, while in the opposite case, $\dot{W} < 0$, we decrease the power input needed for the process.

Variations in $\dot{W}$ can only be achieved, if at least one other quantity is allowed to change in the energy balance Eq. (1). The method of exergy assumes that the system under consideration interacts with a reference environment, which has temperature T_0. For the derivation of the exergy balance one assumes that the heat transfer with the environment $\dot{Q}_0$ exits and is allowed to change, while all other parameters of the process remain unchanged.

Eliminating $\dot{Q}_0$ between the two laws yields for the power

$$\dot{W} = \sum_{k=1}\left(1 - \frac{T_0}{T_k}\right)\dot{Q}_k + \dot{m}(\psi_1 - \psi_2) - T_0\dot{S}_{gen} \tag{2}$$

where we have introduced the specific flow exergy by

$$\psi = h - h_0 + \frac{1}{2}C^2 - T_0(s - s_0). \tag{3}$$

The reference values are chosen in order to make the values of the exergies at atmospheric conditions (T_0, p_0) equal to zero.

Since the entropy production rate $\dot{S}_{gen}$ is strictly positive, the power becomes maximal for vanishing entropy production, i.e., for reversible processes. We define the reversible work for the process by assuming $\dot{S}_{gen} = 0$ as

$$\dot{W}_{rev} = \sum_{k=1}\left(1 - \frac{T_0}{T_k}\right)\dot{Q}_k + \dot{m}(\psi_1 - \psi_2) \tag{4}$$

The irreversibility is defined as

$$\dot{W}_{lost}^{exergy} = \dot{W}_{rev} - \dot{W} = T_0\dot{S}_{gen} \tag{5}$$

This is the power we loose in the actual process, if we compare it to a reversible process between the same end states and exchanging heat with the environment at T_0. If $\dot{W} > 0$, irreversible effects diminish the power output, and if $\dot{W} < 0$ irreversible effects increase the power needed to run the process.

It follows that in order to operate a process more efficiently one should diminish entropy production, if the amount of lost work is considered too large. That is, the engineering task is to compare the lost power $T_0\dot{S}_{gen}$ to the actual power $\dot{W}$ and decide whether a diminishing of the losses is worthwhile. If so, one has to identify the sources of entropy production and minimize these. Exergy and second law efficiency have the goal to quantify the losses, and the second law $(1)_2$ must be used to identify the locations of the largest losses. It is noted that the computation of the losses by the exergy method depends on the environmental temperature T_0.

Similar arguments are used in today's undergraduate textbooks, e.g., Refs. [3,4,6] for the exergetic evaluation of all types of processes and devices. We note that the elimination of $\dot{Q}_0$ in deriving Eq. (2) becomes senseless in the context of an adiabatic device. That is, when $\dot{Q}_0 = 0$, Eq. $(1)_2$ can be multiplied by any temperature and then combined with Eq. $(1)_1$, and consequently the irreversibility can become non-unique.

3. Isentropic efficiency and work loss

Let us consider the expansion of a perfect gas (ideal with constant specific heats) in an adiabatic turbine. The entrance conditions are T_1, p_1 and the gas will expand to local atmospheric pressure p_2. Note that the local atmospheric pressure p_2 in general will be different from the pressure of the reference environment $p_0 = 1.01325$ bar. Ignoring kinetic and potential energies, the first and second laws simply reduce to

$$
\begin{aligned}
w_{\mathrm{T}} &= \frac{\dot{W}}{\dot{m}} = h_1 - h_2 = c_{\mathrm{p}}(T_1 - T_2) \\
\sigma &= \frac{\dot{S}_{\mathrm{gen}}}{\dot{m}} = s_2 - s_1 = c_{\mathrm{p}} \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}
\end{aligned} \tag{6}
$$

We seek the maximum work we can gain from the expansion, but, since we consider an adiabatic turbine, we do not allow for any heat exchange. Here, T_1, p_1, p_2 are fixed while the temperature T_2 is not specified a priori. That is, the temperature T_2 is a natural parameter of the problem. Eliminating T_2 yields

$$w_{\mathrm{T}} = c_{\mathrm{p}} T_1 \left(1 - e^{\sigma/c_{\mathrm{p}}} (p_2/p_1)^{(k-1)/k}\right), \tag{7}$$

where $k = c_{\mathrm{p}}/c_{\mathrm{v}}$.

Since both, σ and c_{p}, are positive, the work output of the turbine becomes maximal for $\sigma = 0$, i.e., the maximum output from an adiabatic turbine (at fixed T_1, p_1, p_2) is given by the isentropic work, which can be expressed as

$$w_{\mathrm{T},s} = c_{\mathrm{p}} T_1 \left(1 - (p_2/p_1)^{(k-1)/k}\right) \tag{8}$$

Then, the temperature of the expanded gas would be given by

$$T_{2s} = T_1 (p_2/p_1)^{(k-1)/k} \tag{9}$$

The actual turbine is irreversible, and is usually characterized by the isentropic efficiency of the turbine, defined as the ratio of the actual work to the isentropic work, viz.

$$\eta_{\mathrm{T}} = \frac{w_{\mathrm{T}}}{w_{\mathrm{T},s}} = \frac{h_1 - h_2}{h_1 - h_{2s}} = \frac{T_1 - T_2}{T_1 - T_{2s}} \leqslant 1 \tag{10}$$

It is clear that a good turbine should have an isentropic efficiency close to unity, and when seeking higher efficiency one should try to construct the turbine accordingly. The irreversible processes in the turbine account for the entropy production σ. The engineering task for improving the turbine efficiency is to diminish the entropy production, i.e., to identify those locations in the turbine which produce most entropy and redesign them.

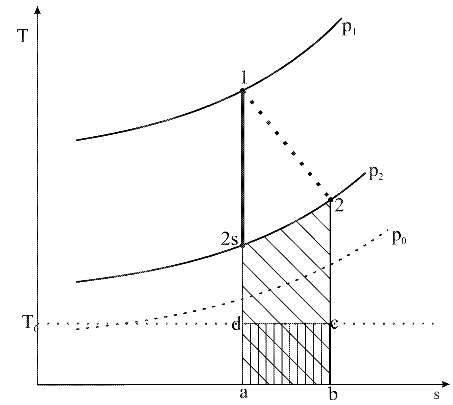

Fig. 1. Adiabatic expansion of ideal gas in T–s diagram.

Note that the isentropic efficiency η_{T} compares the actual process (characterized by T_2) to a hypothetical one with different exit temperature T_{2s}, but with the same type of process, viz. adiabatic expansion in a turbine.

The work loss due to the irreversibilities in the turbine, based on an isentropic approach, is given by

$$
\begin{aligned}
w_{\mathrm{T,lost}} &= w_{\mathrm{T},s} - w_{\mathrm{T}} = h_2 - h_{2s} \\
&= \int\limits_{\substack{2s \\ p=\mathrm{const}}}^{2} \mathrm{d}h = \int\limits_{\substack{2s \\ p=\mathrm{const}}}^{2} T\,\mathrm{d}s
\end{aligned} \tag{11}
$$

The last step of Eq. (11) follows since the Points 2 and $2s$ are at the same pressure, that is the integration is performed at constant pressure. Fig. 1 shows the T–s diagram for the adiabatic expansion of an ideal gas. Here, Point 1 denotes the initial state (T_1, p_1), Point $2s$ denotes the state after the (hypothetical) adiabatic reversible expansion, and Point 2 denotes the actual state after adiabatic irreversible expansion. Thus, the lost work is the hatched area in Fig. 1, with the corner points a, b, 2, $2s$.

For later use, we note that for given turbine efficiency the temperature after the expansion can be written as

$$T_2 = T_1 \left[1 - \eta_{\mathrm{T}} \left(1 - (p_2/p_1)^{(k-1)/k}\right)\right] \tag{12}$$

Also, the expressions for the turbine work and the lost work, respectively, can be written as

$$
\begin{aligned}
w_{\mathrm{T}} &= \eta_{\mathrm{T}} w_{\mathrm{T},s} = \eta_{\mathrm{T}} c_{\mathrm{p}} T_1 \left(1 - (p_2/p_1)^{(k-1)/k}\right) \\
w_{\mathrm{T,lost}} &= (1 - \eta_{\mathrm{T}}) w_{\mathrm{T},s} \\
&= (1 - \eta_{\mathrm{T}}) c_{\mathrm{p}} T_1 \left(1 - (p_2/p_1)^{(k-1)/k}\right)
\end{aligned} \tag{13}
$$

Therefore, for given isentropic efficiency and pressure ratio, the work loss according to Eq. (13) grows with the turbine inlet temperature T_1.

4. Second law efficiency and work loss

Now we want to evaluate the turbine performance by means of the exergy method. For a stationary turbine with inlet conditions (T_1, p_1) and outlet conditions (T_2, p_2), the exergy method gives the maximum work output as

$$w_{\mathrm{T,rev}} = \psi_1 - \psi_2 = h_1 - h_2 - T_0(s_1 - s_2) = w_{\mathrm{T}} + T_0\sigma \tag{14}$$

where T_0 denotes the temperature of the reference environment. We note that the reversible work $w_{\mathrm{T,rev}}$ corresponds to a process which exchanges heat with the environment at T_0, as becomes clear from the derivation of Eq. (4) above.

It follows immediately that the lost work according to the exergy method, relative to any ideal or reversible process between points 1 and 2, is given by

$$w_{\mathrm{T,lost}}^{\mathrm{exergy}} = w_{\mathrm{T,rev}} - w_{\mathrm{T}} = T_0\sigma \tag{15}$$

Therefore, the lost work according to the exergy method is the cross-hatched area in the T–s diagram of Fig. 1, i.e., the rectangle (a, b, c, d).

One commonly used measure of performance in the exergetic interpretation is the second law efficiency, defined as

$$\eta_{\mathrm{II}} = \frac{w_{\mathrm{T}}}{w_{\mathrm{T,rev}}} = \frac{h_1 - h_2}{h_1 - h_2 + T_0\sigma} = \frac{1}{1 + T_0\sigma/(h_1 - h_2)} \leqslant 1 \tag{16}$$

Many other exergy-based and second law-based efficiencies have been developed and applied. Here, however, we focus on the exergy efficiency defined in Eq. (16) as an important illustration. A more comprehensive treatment would require a more exhaustive examination of other exergy efficiencies.

As for the isentropic efficiency η_{T}, the second law efficiency is less than or equal to 1, and can be 1 at best, i.e., when the actual process is reversible, so that $\sigma = 0$.

In order to compare the two efficiencies, η_{T} and η_{II}, we use the constitutive equations of a perfect gas in Eq. (16) and eliminate T_2 by Eq. (12), to obtain

$$\eta_{\mathrm{II}} = \eta_{\mathrm{T}}\left(1 - \left(\frac{p_2}{p_1}\right)^{(k-1)/k}\right)\left[\eta_{\mathrm{T}}\left(1 - \left(\frac{p_2}{p_1}\right)^{(k-1)/k}\right) + \frac{T_0}{T_1}\ln\left[\left(\frac{p_1}{p_2}\right)^{(k-1)/k}(1 - \eta_{\mathrm{T}}) + \eta_{\mathrm{T}}\right]\right]^{-1} \tag{17}$$

This equation gives the second law efficiency η_{II} as a function of the inlet temperature T_1, the turbine pressure ratio p_2/p_1, and the isentropic efficiency η_{T}. Table 1 shows some values of η_{II} as function of η_{T} for a variety of inlet temperatures T_1, a pressure ratio $p_1/p_2 = 10$, $k = 1.4$, and $T_0 = 298.15$ K

For turbines with exit temperatures T_2 above T_0, the numerical value of η_{II} is always larger than the corresponding value of η_{T}, more so for larger inlet temperatures T_1. Indeed, inspection of Eq. (17) shows that η_{II} will go to unity as T_1 goes to infinity, since the factor T_0/T_1 in the denominator will decrease to zero.

For turbines with exit temperatures below T_0 we observe the opposite: the second-law efficiency is smaller than the isentropic efficiency. Indeed, inspection of Eq. (17) shows that η_{II} will go to zero as T_1 goes to zero.

Table 1
Comparison of isentropic efficiency η_{T} and second law efficiency η_{II} for an adiabatic turbine with different values of inlet temperature T_1 and a pressure ratio $p_1/p_2 = 10$. Here, $T_0 = 298.15$ K and $k = 1.4$

η_{T}	η_{II} $T_1 = 1700$ K	η_{II} $T_1 = 1100$ K	η_{II} $T_1 = 575.6$ K	η_{II} $T_1 = 298.15$ K
1	1	1	1	1
0.9	0.965	0.947	0.904	0.830
0.8	0.928	0.892	0.813	0.693
0.7	0.887	0.835	0.726	0.578
0.6	0.839	0.771	0.638	0.478
0.5	0.782	0.700	0.549	0.387
0.4	0.712	0.616	0.456	0.303

The influence of the temperature between the two approaches, exergetic and isentropic, becomes also apparent when we study the work losses, i.e., the areas $A_s = [a, b, 2, 2s]$ and $A_{\mathrm{ex}} = [a, b, c, d]$ in Fig. 1: When we express the entropy production $\sigma = s_2 - s_1$ as a function of the isentropic efficiency and the pressure ratio by means of Eqs. (12) and (6) we obtain

$$w_{\mathrm{T,lost}}^{\mathrm{exergy}} = c_{\mathrm{p}} T_0 \ln\left[(p_1/p_2)^{(k-1)/k}(1 - \eta_{\mathrm{T}}) + \eta_{\mathrm{T}}\right] \tag{18}$$

Therefore, for given isentropic efficiency and pressure ratio, the work loss according to the exergy method is independent of the turbine temperature. From the figure, it becomes evident that the exergetic work loss will be smaller than the isentropic work loss that we computed for the adiabatic turbine, expressed in Eq. $(13)_2$, as long as the exit temperature of the turbine is above T_0.

5. Lost work and recoverable work

In the last section we saw that the two measures for turbine performance, isentropic efficiency and second-law efficiency, can differ considerably. Accordingly, an engineer who is asked to evaluate the performance of a turbine might be rather confused. In this section, we therefore try to explain the differences between the two approaches.

We learned that the second law efficiency η_{II} of an adiabatic turbine normally is larger than the isentropic efficiency η_{T}. In particular, we saw that for a given isentropic efficiency η_{T} the second law efficiency η_{II} increases with increasing inlet temperature of the turbine. Also, the lost work according to the exergy method is independent of the turbine inlet temperature, see Eq. (18). This observation leads Kotas to the conclusion “that a turbine stage with a low isentropic efficiency is more tolerable at a high temperature than at a low temperature” [5].

On the other hand, the isentropic lost work, as computed for the adiabatic turbine in Eq. (13), increases with increasing turbine inlet temperature. This might lead to just the opposite interpretation, that is a turbine stage with low isentropic efficiency might be less tolerable at higher temperatures.

5.1. How to realize the maximum work

In order to understand the differences between the two approaches to turbine efficiency, we ask how the reversible work can be realized. This question is normally not addressed in standard undergraduate textbooks, e.g., [3,4,6].

We begin with the discussion of the reversible work related to the isentropic efficiency $w_{T,s}$, Eq. (8). Here, the above question is easily answered: $w_{T,s}$ is the work of an adiabatic reversible, or isentropic, turbine. In order to realize $w_{T,s}$ in a turbine, one has to minimize the irreversibilities in the interior of the turbine, i.e., the entropy production σ.

For the reversible work of the exergy method $w_{T,\mathrm{rev}}$, Eq. (14), the answer is that any reversible process between the Points 1 and 2 will give the reversible work $w_{T,\mathrm{rev}}$, as long as all heat transfers to the environment take place reversibly, i.e., through a (infinite) series of Carnot engines. Indeed, the first law for a flow process between 1 and 2 reads

$$w_{12}^{\mathrm{flow}} = h_1 - h_2 + q_{12} \tag{19}$$

where w_{12}^{flow} is the work and q_{12} is the heat exchange for the process. Since we are considering a reversible process, we have $q_{12} = \int T\,\mathrm{d}s$. In particular, $T\,\mathrm{d}s$ is the heat transfer at temperature T and this can drive an infinitesimal Carnot engine with efficiency $\eta_C = 1 - T_0/T$ rejecting heat to the environment. It follows, that the work

$$\begin{aligned} w_{12}^{C} &= -\int_1^2 \left(1 - \frac{T_0}{T}\right) T\,\mathrm{d}s \\ &= -\int_1^2 (T - T_0)\,\mathrm{d}s = T_0\sigma - q_{12} \end{aligned} \tag{20}$$

can be gained through the Carnot engines. The total work is given by

$$w_{12}^{\mathrm{flow}} + w_{12}^{C} = h_1 - h_2 + T_0\sigma = w_{T,\mathrm{rev}} \tag{21}$$

The simplest process of this kind, and the only one which does not employ external Carnot engines, follows the curve [1-d-c-2] in the T–s-diagram of Fig. 1:

1-d isentropic expansion from (T_1, p_1) to (T_0, p_d)
d-c isothermal heat supply from (T_0, p_d) to (T_0, p_c)
c-2 isentropic compression from (T_0, p_c) to (T_2, p_2)

Of course, the process details depend on the choice of T_0. If $T_0 \geqslant T_2$, the last step will be an expansion instead of a compression. For the usual conditions in a gas turbine system we have $T_0 \leqslant T_2$ and the process is as described above.

It is no surprise that the process involves heat transfer at T_0, since the derivation of the expression for the reversible work allowed explicitly for an exchange of heat with the environment at T_0.

The point which we wish to emphasize here is that a reversible process between Points 1 and 2 cannot be realized by an adiabatic turbine alone. By using the exergy method for assessment of our actual process, we compare two different processes, which differ not only in the fact that one is reversible (1-d-c-2) and one is not (1-2), but also in the details of the processes—the hypothetical process is much more involved, in the simplest case adding heat exchanger (c-d) and compressor (c-2) to the plain expansion (1-2, and 1-a, respectively).

5.2. The recoverable heat of friction

There is another interpretation of the second law efficiency and the corresponding work losses as the recoverable heat of friction. In his book [5], Kotas writes: “Because of frictional reheat, the enthalpy and the exergy of the working fluid in the final stage of the actual process are greater than they would have been under isentropic conditions. When the final state of an expansion process corresponds to the initial state of another process, e.g., in multistage turbines, this difference in enthalpy or exergy can be utilised. Consequently, we must not regard the whole of the frictional reheat as a loss.”

In the following, we shall try to interpret this statement for a single turbine. Multi-stage turbines will be discussed elsewhere [11].

The final state of an isentropic turbine is the Point $2s$ (see Fig. 1). This state has some exergy, i.e., work potential, ψ_{2s} so that the exhaust has some value and could be used to produce work. The exhaust of the actual turbine has exergy $\psi_2 > \psi_{2s}$, i.e., it has a greater work potential. This difference in exergies gives us the maximum work for a process [2-2s] as

$$\begin{aligned} w_{2,2s}^{\mathrm{rev}} &= \psi_2 - \psi_{2s} = h_2 - h_{2s} - T_0(s_2 - s_{2s}) \\ &= \int_2^{2s} T\,\mathrm{d}s - T_0(s_2 - s_{2s}) = \mathrm{Area}[2s, d, c, 2] \end{aligned} \tag{22}$$

See Fig. 1 for a visualization of the work $w_{2,2s}^{\mathrm{rev}}$ in terms of an area in the T–s-diagram. If we accept the irreversibility of the turbine between 1 and 2, we can nevertheless get some work out by a process to Point $2s$. The maximum work for the latter is $w_{2,2s}^{\mathrm{rev}}$. In other words, we can add some devices (the simplest being reversible isobaric cooling through Carnot cycles) to the irreversible turbine, and achieve the work

$$w_{1\text{-}2\text{-}2s}^{\max} = w_T + w_{2,2s}^{\mathrm{rev}} = h_1 - h_{2s} - T_0\sigma = w_{T,s} - T_0\sigma \tag{23}$$

This is the maximum work for the process [1-2-2s], which includes the irreversible subprocess [1-2].

For the interpretation of the above results, we recall that the work loss of the irreversible turbine in comparison to the isentropic turbine is given by Eq. (11). Therefore, we can rewrite Eq. (22) as

$$w_{2,2s}^{\mathrm{rev}} = w_{\mathrm{T,lost}} - T_0\sigma \tag{24}$$

and it follows that we can recover the part $w_{2,2s}^{\mathrm{rev}}$ of the work lost in the turbine, while the remainder $T_0\sigma$ is lost indeed, and not recoverable. Accordingly, the work loss must be compared to the reversible turbine which also has endpoint $2s$, and is given by

$$w_{1\text{-}2\text{-}2s}^{\mathrm{lost}} = w_{\mathrm{T},s} - w_{1\text{-}2\text{-}2s}^{\max} = T_0\sigma \tag{25}$$

Note, that these considerations lead to the definition of yet another efficiency

$$\eta_{\mathrm{new}} = \frac{w_{1\text{-}2\text{-}2s}^{\max}}{w_{\mathrm{T,rev}}} = 1 - \frac{T_0\sigma}{h_1 - h_{2s}} \leqslant 1 \tag{26}$$

which measures the work output of the irreversible turbine plus recovery device against the isentropic turbine.

5.3. *Lost work, again*

We have now obtained three different expressions for the lost work:

- The lost work for the irreversible turbine [1-2] compared with the corresponding isentropic turbine [1-2s] as expressed by Eqs. (11), (13)

  $$w_{\mathrm{T,lost}} = h_2 - h_{2s} = c_{\mathrm{p}} T_{2s}\left[e^{\sigma/c_{\mathrm{p}}} - 1\right]$$

  For comparison with other results from the exergy method, one can write

  $$w_{\mathrm{T,lost}} = T^*\sigma \quad \text{where } T^* = \frac{c_{\mathrm{p}}}{\sigma} T_{2s}\left[e^{\sigma/c_{\mathrm{p}}} - 1\right]$$

  For comparatively small values of the entropy production, the exponential can be expanded to first order so that $T^* = T_{2s}$.
- The lost work for the irreversible turbine [1-2], compared to the maximum work of a reversible process [1-2] as expressed by Eq. (18)

  $$w_{\mathrm{T,lost}}^{\mathrm{exergy}} = T_0\sigma$$

- The lost work for the turbine plus recovery device, i.e., the process [1-2-2s], compared to the isentropic turbine [1-2s] as expressed by Eq. (25),

  $$w_{1\text{-}2\text{-}2s}^{\mathrm{lost}} = T_0\sigma$$

Surprisingly, the lost work expressions for the last two cases are equal $(= T_0\sigma)$. Therefore we emphasize that the two lost work expressions, although equal in size, refer to different processes, viz. a reversible process [1-2], and a partly irreversible process [1-2-2s], respectively.

6. Conclusions

In this article, we show that the isentropic efficiency for a turbine and a particular turbine exergy efficiency in general can differ, sometimes significantly. Correspondingly, the work lost in a turbine, which is essentially the difference between the actual and the ideal turbine work outputs, also differs when isentropic efficiency and exergy efficiency approaches are taken. The results are not necessarily surprising, in that these efficiencies measure different quantities and therefore address different questions. However, this situation can confuse potential users, especially if they do not carefully assess the nature of the different efficiency definitions before applying them. We believe that the explanations and discussions presented here can eliminate or reduce the possible confusion that this situation can create for users, and also leave engineers better aware of what steps are needed, based on thermodynamic assessments, to improve efficiency, and of what the true ideal efficiency that they face is.

In particular, it becomes clear that there are two possibilities to reduce the lost work of a turbine:

(a) reduce the irreversibilities inside the turbine, and
(b) add recovery devices to the turbine.

From our analysis we conclude that the isentropic efficiency is the proper measure when one considers the turbine *alone*. The consideration of the isentropic efficiency teaches us to diminish the interior irreversibilities of the turbine as much as possible. However, the isentropic analysis does not take into account that parts of the losses can be recovered by adding additional devices.

While the second law efficiency increases with the work potential of the turbine exhaust, it does not rely on accounting for a recovery device for exhaust exergy, but on replacing the irreversible turbine by a more complicated reversible process. The reversible process (1-2) is as difficult to achieve as the reversible turbine (1-2s), and therefore it remains unclear what engineering action follows from its use.

The newly introduced efficiency (26) considers the irreversible turbine and an additional exhaust recovery device. While we do not propose that this efficiency is the better choice, we introduce it to emphasize that the best improvement of a turbine relies on a combination of diminishing the internal irreversibilities and recovering the heat of friction from the exhaust.

All three efficiencies are equal to unity in the case of a reversible turbine, and the corresponding work loss is zero. A possible conclusion from this observation is that the diminishing of the internal irreversibilities leads to a greater improvement than the recovery of heat of friction.

Although this article applies to a simple process, it can likely be extended in some ways to other situations. In general, we believe this article serves to illustrate the need to use caution when defining and applying efficiencies, for turbines and likely also for other devices. It is not sufficient simply to understand and appreciate the benefits of efficiencies based on the second law compared to those based on the first law, although this is very important. At the same time, one has to understand what suggestions to improve the process at hand can be deduced from the analysis. Only a complete understanding of both will allow engineers to utilize the different turbine efficiencies more effectively, and facilitate appropriate and beneficial uses of exergy methods.

A statement, sometimes found in books about exergy, is that work losses at high temperatures are less important than those at low temperatures. Accordingly, irreversibilities in the high-pressure regions of a turbine could be construed to be less important than those in the low-pressure regions [2, 5]. Yet an analysis based on isentropic efficiencies shows that irreversibilities at any pressure contribute to the overall performance of the turbine with the same weight. This statement will be proven in a forthcoming paper [11].

Acknowledgements

The authors are grateful to Prof. J.B. Haddow for valuable discussions. This research was supported by the Natural Sciences and Engineering Research Council of Canada.

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